Detecting Extreme Mass-Ratio Inspirals using Time-Frequency Method

Linqing Wen

Max-Planck Institut fuer Gravitationsphysik
Albert Einstein Institut
Germany

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In collaboration with

Jonathan Gair, Yanbei Chen, Stas Babak, et al.

Outline

- Characteristic of EMRIs
- Time-Frequency detection method
- Performance
- Discussion
  - Confusion problem
  - Parameter estimation

Extreme Mass-Ratio Inspirals (EMRIs)

- Typical systems :
  - white dwarfs, neutron stars, and stellar-mass black holes (0.6-50 Msun) onto $10^5 - 5\times10^6$ Msun super-massive Black Holes.
- Large parameter space
  - ~14 parameters, 7 intrinsic
  - Spin S, and eccentricity are important

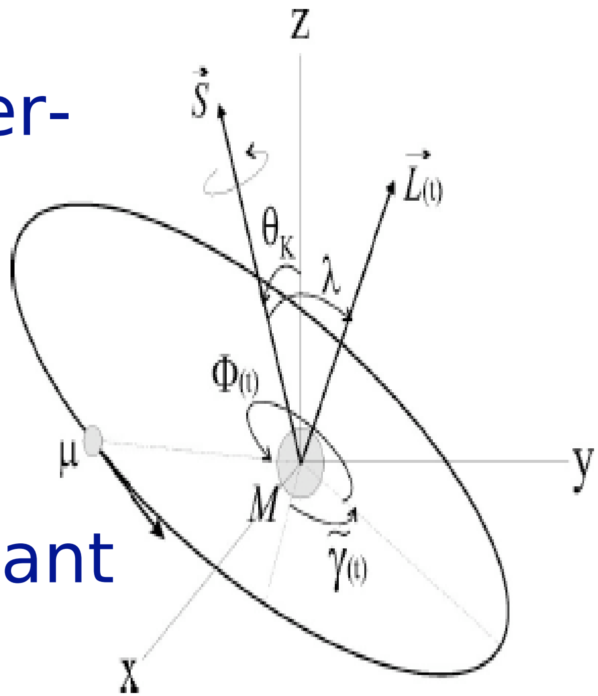


Fig. from Barack & Cutler 2004

$$M, \mu$$

$$(\ S, \lambda_{LS}\ ), e_{0,}\ percenter\ \gamma_{0,}\ initial\ phase\ \phi_0$$

Extreme Mass-Ratio Inspirals (EMRIs)

- Event rate is high
  - Estimated number of EMRI events is high

    (Gair, L. 2004; LIST Report, Barack L. et al.)
  - The event rate can be ~1000 in 3-5 years within ~ 3.5 Gpc for $10+10^6 Msun$ systems.
  - Problem with identifying events
    - galactic WD-WD binary and possibly EMRI background

EMRIs: data analysis perspective

- rms SNR at each frequency is small
  - Typically <0.1

$$h \sim 6 \times 10^{-22} \left(\frac{r}{Gpc} \right)^{-1} \left(\frac{M}{10^6 Ms} \right)^{2/3} \frac{\mu}{10\, M_s} \left(\frac{f}{5\, mHz} \right)^{2/3}$$

- Detections based on simple Fourier transform are generally not possible

EMRIs: data analysis aspect

- Merging frequency scaled inversely with mass

$$f_M \sim \frac{4.4}{M/10^6 M_S} \quad mHz$$

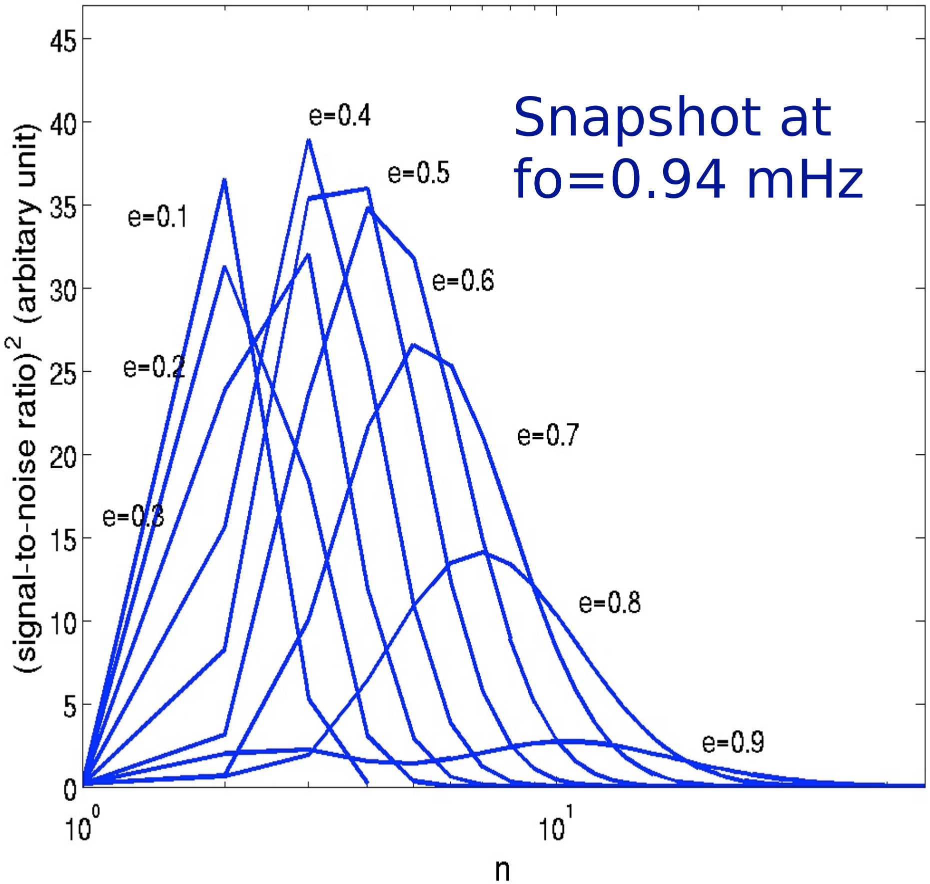
- Large number of frequency bins

$$N_f \sim f_M/df \sim 5\times 10^5 \quad (for\, T=3\, yr,\, f_m=5\, mHz)$$

$$SNR(d\sim 1\, Gpc)\sim 100$$

Extreme Mass-Ratio Inspirals (EMRIs)

$$SNR_n^2 \propto \frac{\dot{E}(f_n)}{\dot{f}_n f_n S_h(f_n)} \Delta(\ln f_n)$$



Orbits are typically eccentric

- e ~0.1-0.7, signal power spreads into many harmonics
- At e>0.1, especially at low-f, SNRs at higher harmonics become important due to noise response

Extreme Mass-Ratio Inspirals (EMRIs)

- Complicated waveform
  - Three Characteristic frequencies
    - radial frequency
    - GR periastron precession
    - orbital plane precession from S-L coupling
  - Modulation from LISA's orbital motion
    - Amplitude
    - Frequency

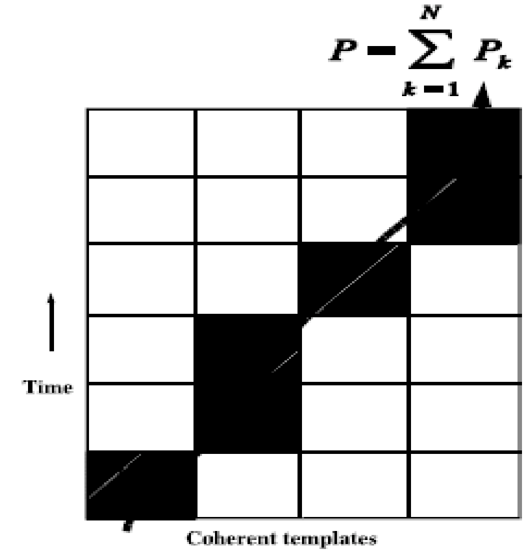
(Cutler 98, Barack & Cutler 2004)

EMRIs: Computational Challenge

- Fully coherent detection is impossible
  - Waveform can eventually be calculated
  - Optimal: best in MLR/SNR
  - impossible computational cost
    - $\sim 10^{30} - 10^{40}$ templates needed for fully coherent search of 3 year of data
    - $\sim 10^{12}$ templates/yr possible for 50 Tflop computer cluster ....

  (Cutler's talk, Gair, L. et al 2004, LIST Report)

EMRIs: Alternative Method



- Semi-coherent method
  - Search segments of data coherently but add incoherently
    - Search for 10^{10} templates coherently
      - 2wk coherent search for 3 yr data
      - Use all available computer power
      - Then add powers along ~1e5 tracks
    - SNR required increased by a factor of ~2 from full coherent one at FAP~ 0.01

(Cutler’s talk. Gair et al. 2004)

EMRIs: Time-Frequency Methods

- incoherent method
  - No templates
  - Search for maximum power density in t-f plane
    - Windowed FFT for every two weeks' data
    - For each point in t-f plane, given a rectangular box, calculate total power weighted by noise

$$P(i,k) = 2\left|h_k^i + n_k^i\right|^2 / \sigma_{ik}^2$$

$$\rho(i,k) = \sum_{a=i-n/2}^{a=i+n/2} \sum_{b=k-l/2}^{b=k+l/2} P(i,k)$$

$$< \rho(i,k) > = \rho_{MF} + 4m$$

EMRIs: Time-Frequency Methods

- Robust/popular
  - Signal increases by a factor ~N, noise by sqrt(N)
  - widely used in X-ray astronomy
    - e.g., search for kilo-hertz QPOs in LMXBs
  - Density mapping method is the same as in cosmological N-body simulation to pick out clusters
  - In LIGO data analysis, it is called "excess power" method
  - Simple and fast
    - run-time~ minutes, Matlab code: tens of lines

Simulated Waveforms

- Kludge Waveform

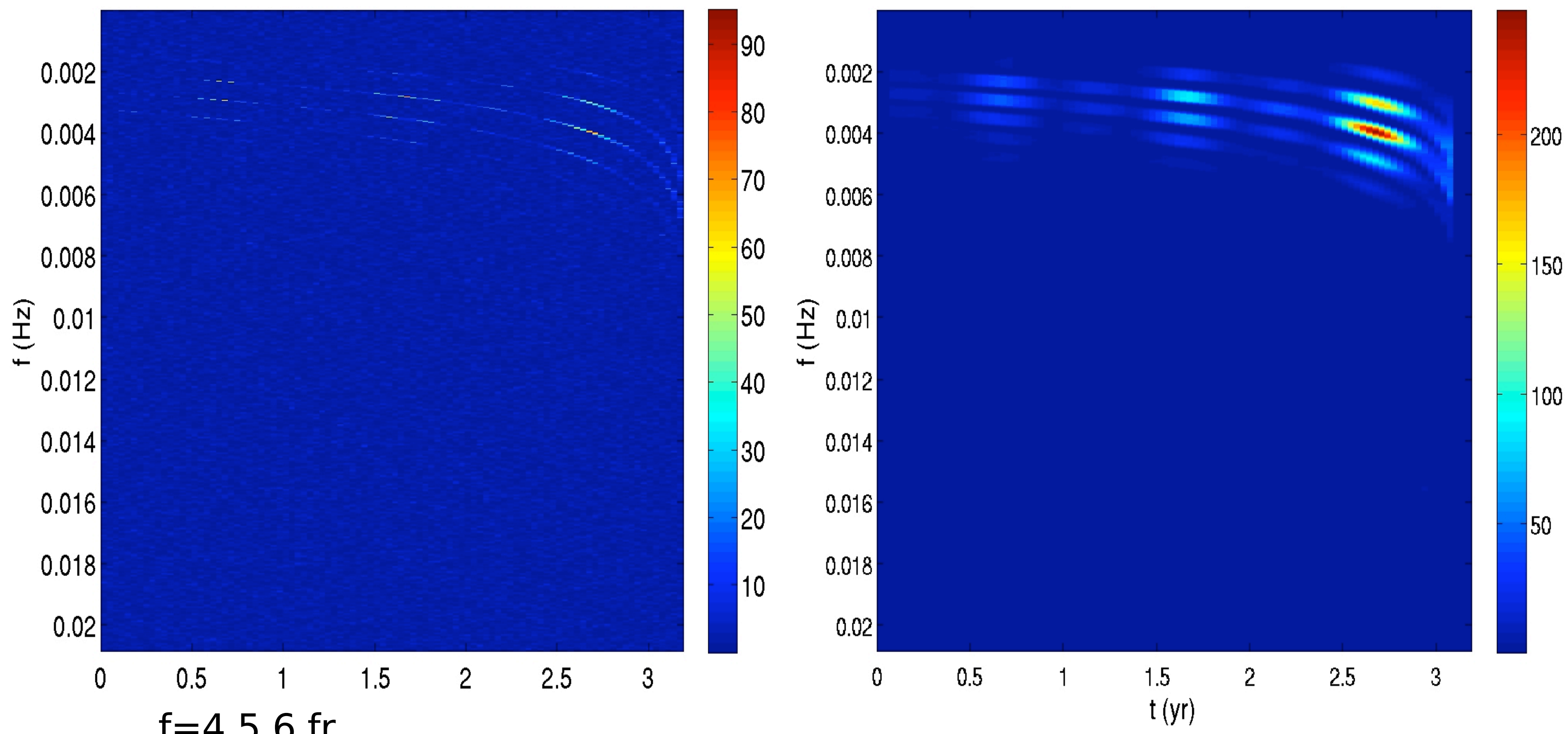
  - Solve exact Kerr geodesic equations
  - PN formula to evolve conservative quantities
  - Quadrupole GW waveform
  - Convolved with LISA orbital amplitude and Doppler modulation
  - Glampedakis, Hughes, Kennefick (2002)

    Gair et al (2005)

Ilustration of a bright 10+1e6 Msun EMRI in t-f plane (at 250 Mpc)

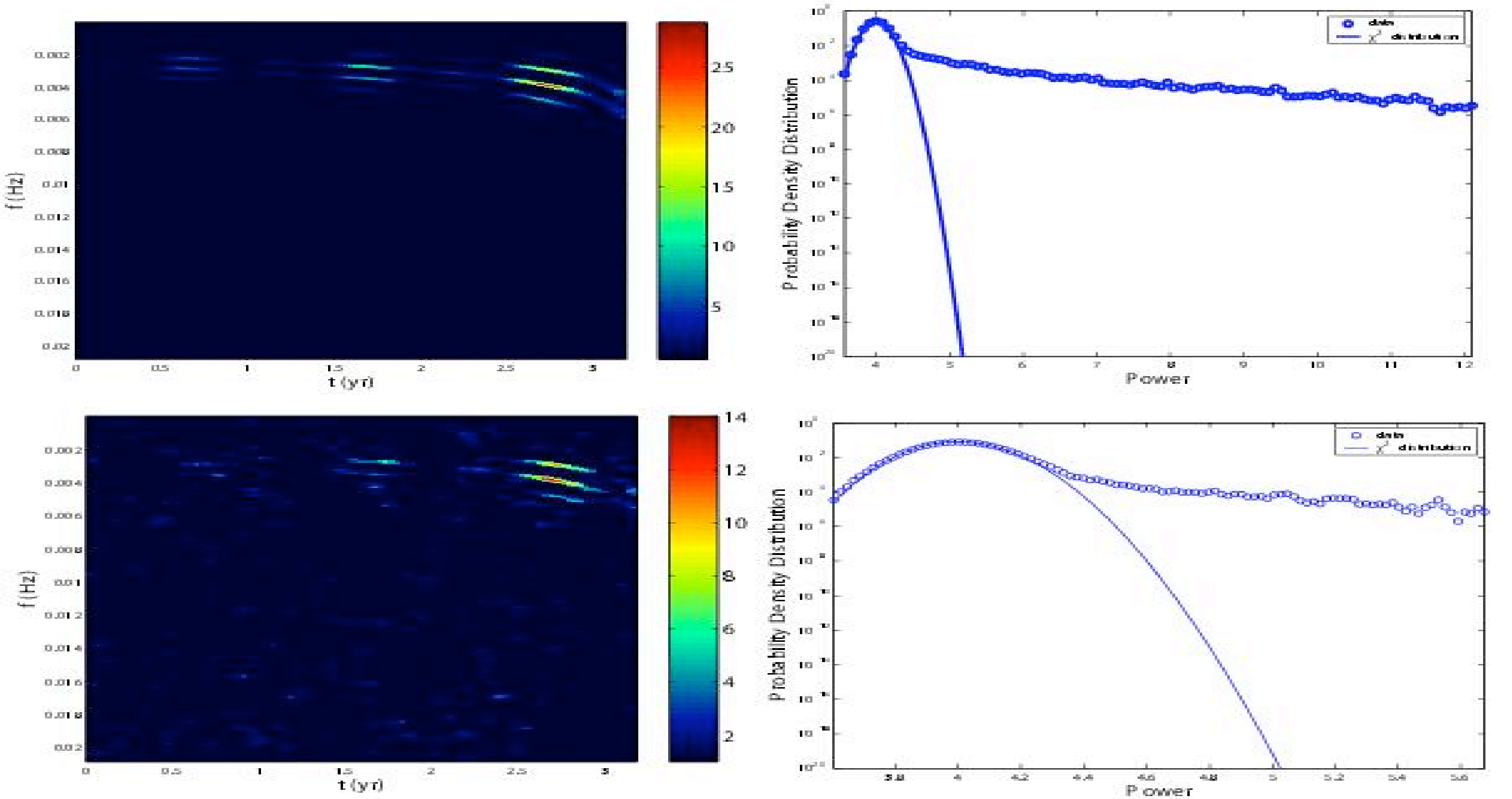
“Raw” T-F Powers

T-F Power density SNR



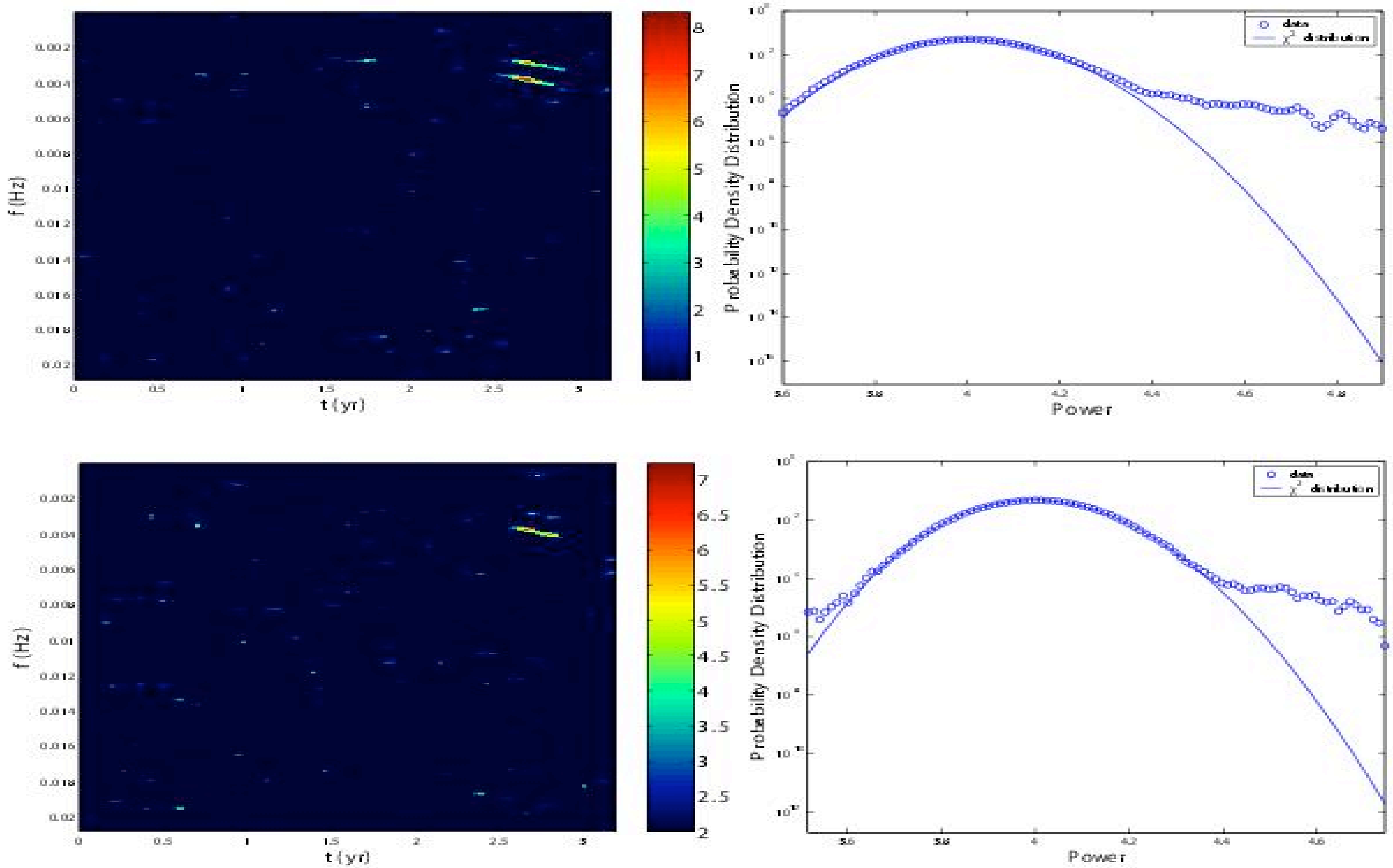
f=4,5,6 fr

EMRIs: Time-Frequency Methods (typical case at 0.5, and 1 Gpc)



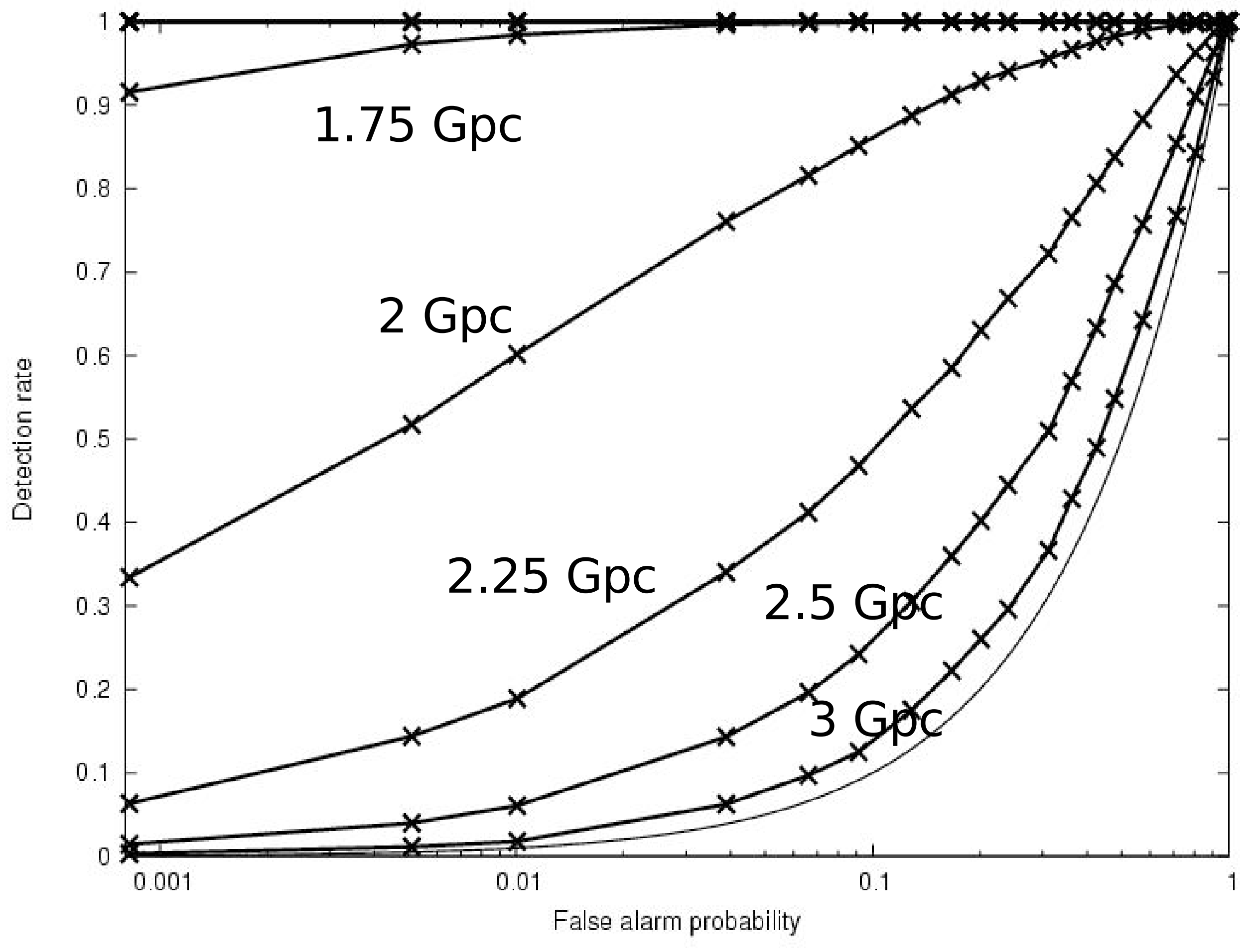
M=1e6+10, e0=0.4, a=0.8

EMRIs: Time-Frequency Methods
(typical case at 1.4, 2 Gpc)



M=1e6+10, e0=0.4, a=0.8

Monte-Carlo Results:
Detection Rate vs False Alarm Probability



At d=2 Gpc, FAP =0.01, detection rate ~ 60%

EMRIs: Time-Frequency Methods

- Typical case, reach 2 Gpc for FAP=0.01

  - Detection based on power density from ~2 wks' data, ~0.2 mHz frequency band

    (Wen & Gair 2005)

  - Monte-Carlo simulation performed for 26 possible systems (different M, m, S, e0, i, theta_s, phi_s, theta_k, phi_k)

    – Detected to 1-3 Gpc, smaller f_dot is better

    (Gair & Wen 2005)

Label	Parameters	Initial p/M	SNR
A	See text	10.3	155
B	$M = 3 \times 10^5 M_\odot$	18.25	119
C	$M = 3 \times 10^6 M_\odot$	6.5	110
D	$m = 0.6 M_\odot$, $M = 3 \times 10^5 M_\odot$	9.405	14.1
E	$m = 0.6 M_\odot$	5.83	21.0
F	$m = 0.6 M_\odot$, $M = 3 \times 10^6 M_\odot$	4.511	15.7
G	$m = 100 M_\odot$	17.78	382
H	$a = 0.95M$	10.07	170
I	$a = 0.5M$	10.74	132
J	$a = 0.1M$	11.31	108
K	$e_0 = 0$	10.42	147
L	$e_0 = 0.1$	10.41	150
M	$e_0 = 0.25$	10.385	151
N	$e_0 = 0.7$	9.71	159
O	$\iota = 0$	9.925	223
P	$\iota = 30^\circ$	10.1	189
Q	$\iota = 60^\circ$	10.59	115
R	$\iota = 120^\circ$	12.126	57.7
S	$\iota = 150^\circ$	12.82	79.6
T	$\iota = 180^\circ$	13.11	87.0
ExtrinsA	$\cos(\theta_S) = 0.99$	10.3	117
ExtrinsB	$\cos(\theta_S) = 0.01$	10.3	162
ExtrinsC	$\cos(\theta_K) = 0.99$	10.3	118
ExtrinsD	$\cos(\theta_K) = 0.01$	10.3	146
ExtrinsE	$\phi_K = 0.01$	10.3	111
ExtrinsF	$\phi_K = 2.$	10.3	111

Table 1. Parameters and signal to noise ratios at 1 Gpc for trial waveforms. Unspecified parameters are the same as source "A", as given in the text.

	0.8 Gpc	1.2 Gpc	1.4 Gpc	2 Gpc	3 Gpc
A	1	1	1	0.60	0.02
B	1	1	0.93	0.04	0.01
C	1	1	1	0.10	0.02
D	0.00	0.00	0.02	0.01	0.01
E	0.01	0.00	0.00	0.02	0.00
F	0.03	0.02	0.02	0.01	0.02
G	1	1	1	1	1
H	1	1	1	0.96	0.01
I	1	1	1	0.17	0.00
J	1	1	0.85	0.02	0.01
K	1	1	1	1	0.51
L	1	1	1	1	0.29
M	1	1	1	1	0.07
N	1	1	0.99	0.22	0.00
O	1	1	1	1	0.63
P	1	1	1	1	0.10
Q	1	1	0.85	0.03	0.00
R	0.8	0.02	0.04	0.00	0.01
S	1	0.53	0.1	0.02	0.02
T	1	0.96	0.36	0.01	0.01
ExtrinsA	1	1	0.65	0.02	0.01
ExtrinsB	1	1	1	0.94	0.03
ExtrinsC	1	1	0.82	0.03	0.02
ExtrinsD	1	1	1	0.31	0.02
ExtrinsE	1	0.99	0.57	0.03	0.02
ExtrinsF	1	0.99	0.62	0.02	0.02

Table 2. Detection rates for trial waveforms at various distances. Thresholds were set using the numerical probability distributions, and with an overall search false alarm probability of 1%.

EMRIs: Time-Frequency Methods
Comparison to (semi) coherent Method

- Required SNR at the same FAP

$$\rho_{TF} \sim (m)^{1/4} \rho^{1/2}$$

$$\rho(FAP \sim 1\%) \sim 15$$

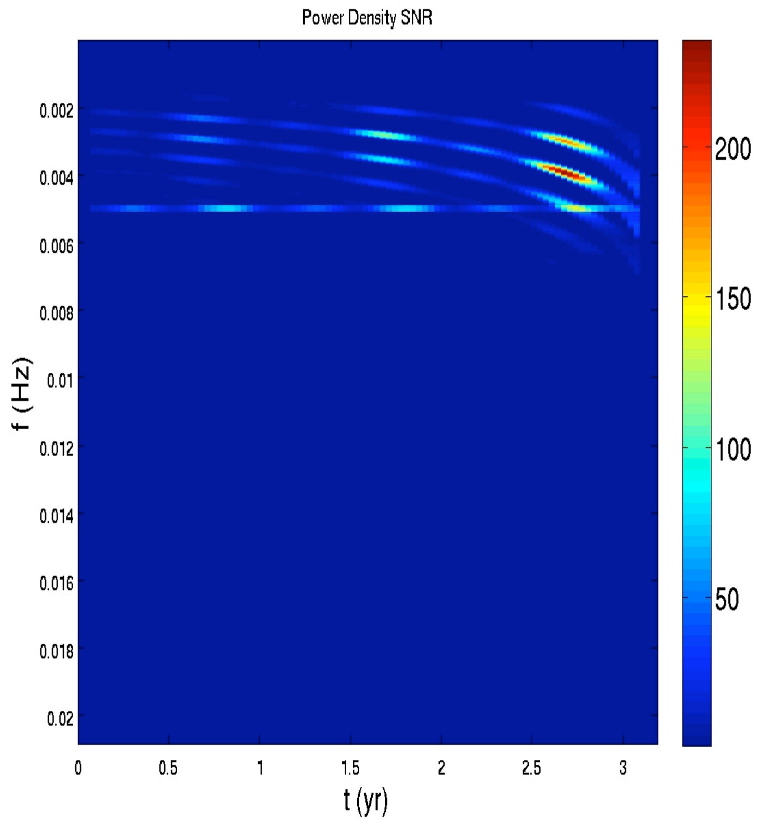
- Best case: m=1, signal concentrates in one t-f bin
  - e.g., sinusoidal signals/WD-WD inspirals
- Worst case, signal spreaded into all bins
- Performance is source dependent
  - Semi-coherent method searches
    - ~ 1e15 templates -> larger FAP
  - T-f method is incoherent
  - but much smaller numbers of searches

Improvement on Detection

- So far detections are based on max. of one blob
  - Works very well
    - Detection EMRIs up to 1-3 Gpc
- Improving t-f method
  - Important to find the track
    - summing all powers on track
  - Worthwhile to search through directions
    - Take care of some confusion sources
  - Including known info of waveform

WD-WD Confusion Problem

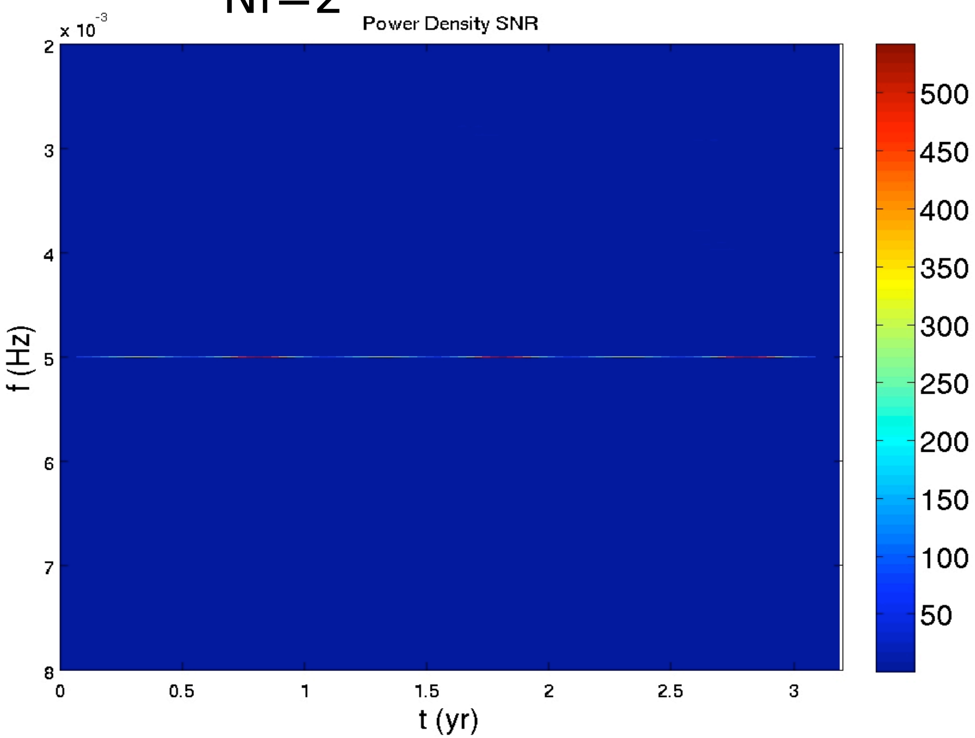
- Can always apply other technique to remove binaries
  - e.g., g-clean, Radon transform, MCMC
- Information extracted from t-f method:
  - Frequency/time spread
  - Directional information
  - T-f Track -> <f(t)>
    - curved track vs straight ones
  - Power-> <dE(t)/dt>



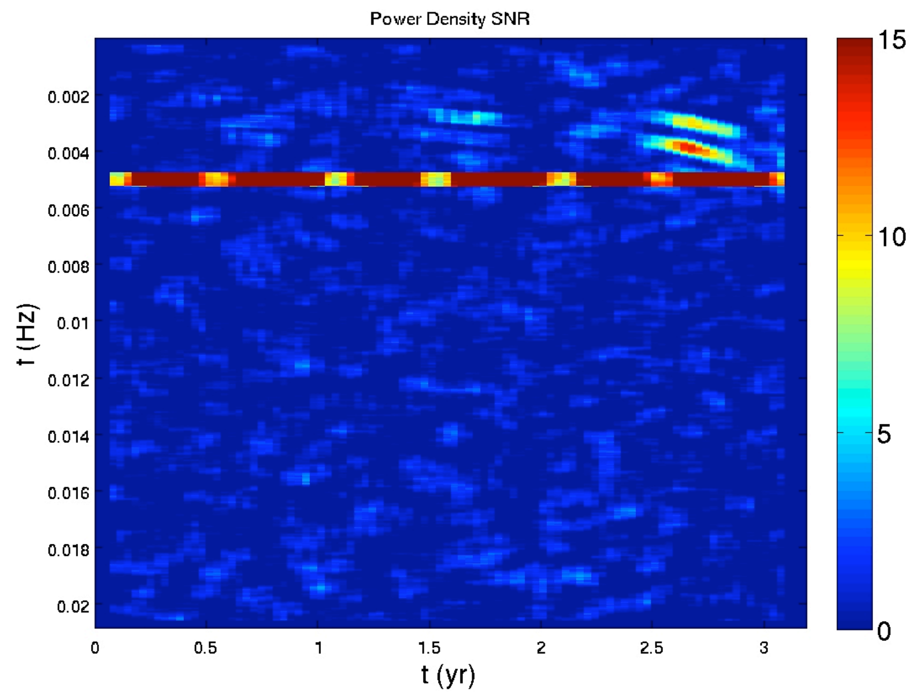
1. Zoom-in with different t-f boxsizes

- Calculate power density with different boxsizes
- For different types of signal, max SNR most likely occurs at different boxsizes = camera zoom-in

Nf=2



Nf=10000 df=8e-7 Hz

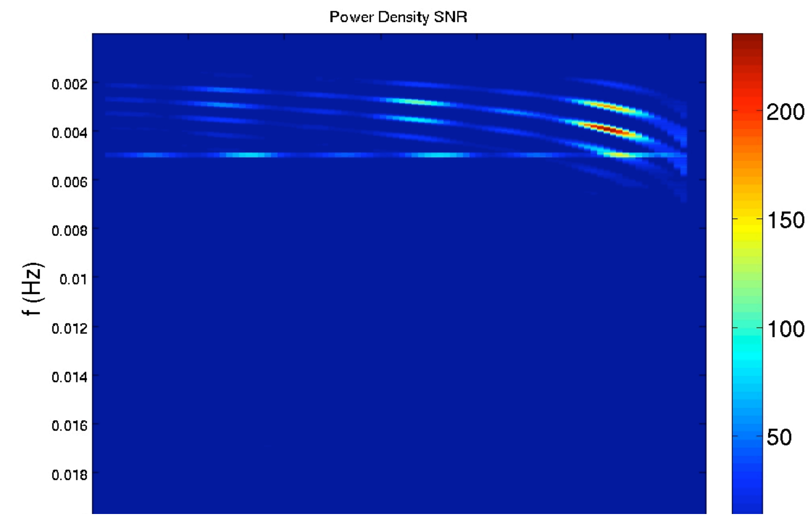


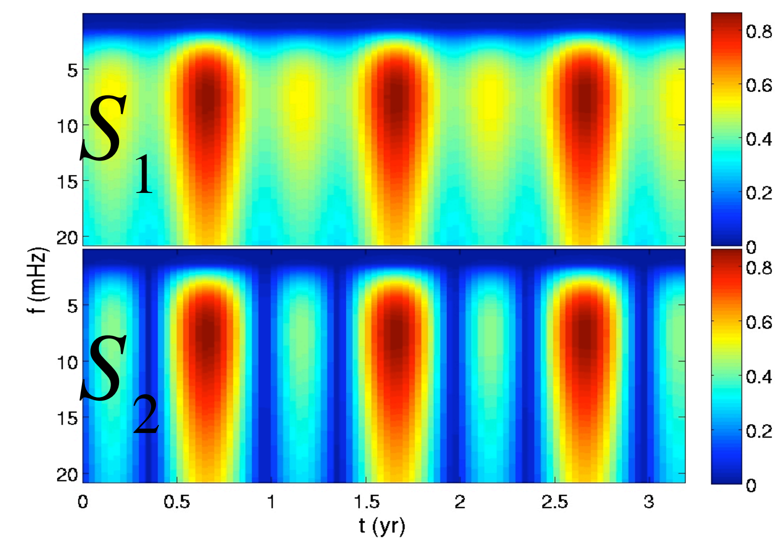
2. Decoding Directional Information

$$SNR^2 = (A\vec{h} \mid A\vec{h}) = s_1^2 h_1 + s_2^2 h_2$$

$$\vec{h}^T = v^T (h_+, h_x)^T, \quad v^T v = I$$

- Detections sensitivity can be ranked by (s1, s2)
  - Red region are more sensitive to GW signals
  - powers from these "designated" area should be added with priority





0-th order approximation(Cutler 98

Decoding directional information

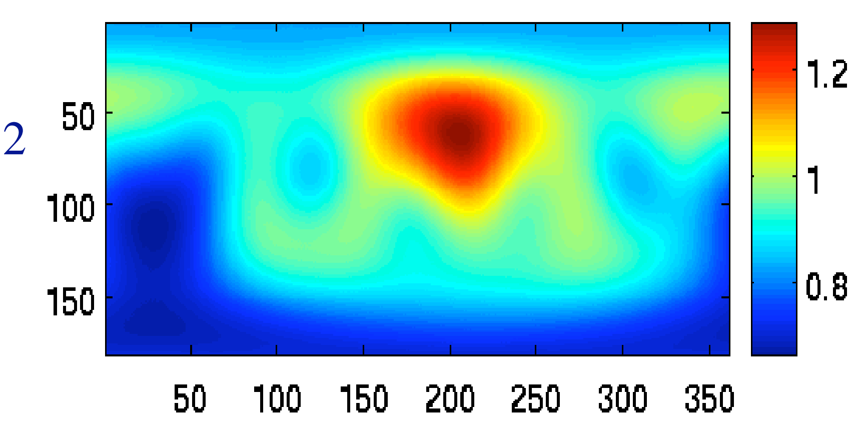
- Realistic LISA configuration encodes more directional information
- At higher f, it is equivalent to 3 detector-network
- high power at each source direction
- corresponding low power in null-stream of that direction (high f)

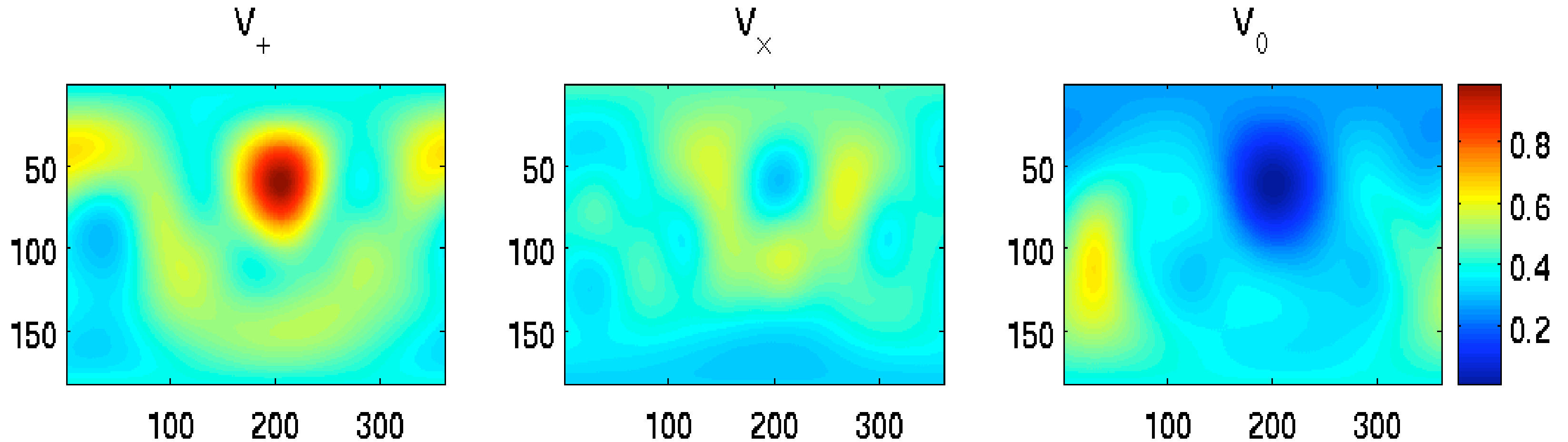
For one source direction

$$SNR^2 = (A\vec{h} \mid A\vec{h}) = s_1^2 h_1 + s_2^2 h_2$$

$$\vec{h}^T = v^T (h_+, h_x)^T, \quad v^T v = I$$

SNR1+SNR2





SNR1 SNR2 Null Stream

See Archana Pai's Poster

F=25 mHz

- Src from different direction has its own bright “blob” and dim spot in null-stream all sky map
- Brighter ones are selected first
- Worthwhile to search over directions

Considerations on Parameter Estimation

- t-f method provided data points

$$track \text{->} f_d(t_i),\ powers \text{->} dE/dt(t_i) \quad (i=1,N) \quad (averaged)$$

- Also provide information on time-frequency spread of powers, harmonics/beats
- 2N data
- Assuming we know the relations between fn
- In case of PN formula, need to fit N+Nc parameters
  - In principle, just least-square fit parameters if N>Nc above threshold
  - Nc~7 $\quad e(t_i), \quad (i=1,N), M, \mu, S\cos\lambda, n, \ldots,$
- For multi-EMRIs: also least-square fit

Conclusion

- Time-frequency method works pretty well
  - As the 1st step of the hierarchical search
- Current implementation can be further improved in detection/confusion problem
  - By finding the tracks
    - e.g., Hough transform
  - By search over source directions
- Parameters can be estimated/constrained
    - Need information that represent dominating f, and dE/dt in an averaged sense,

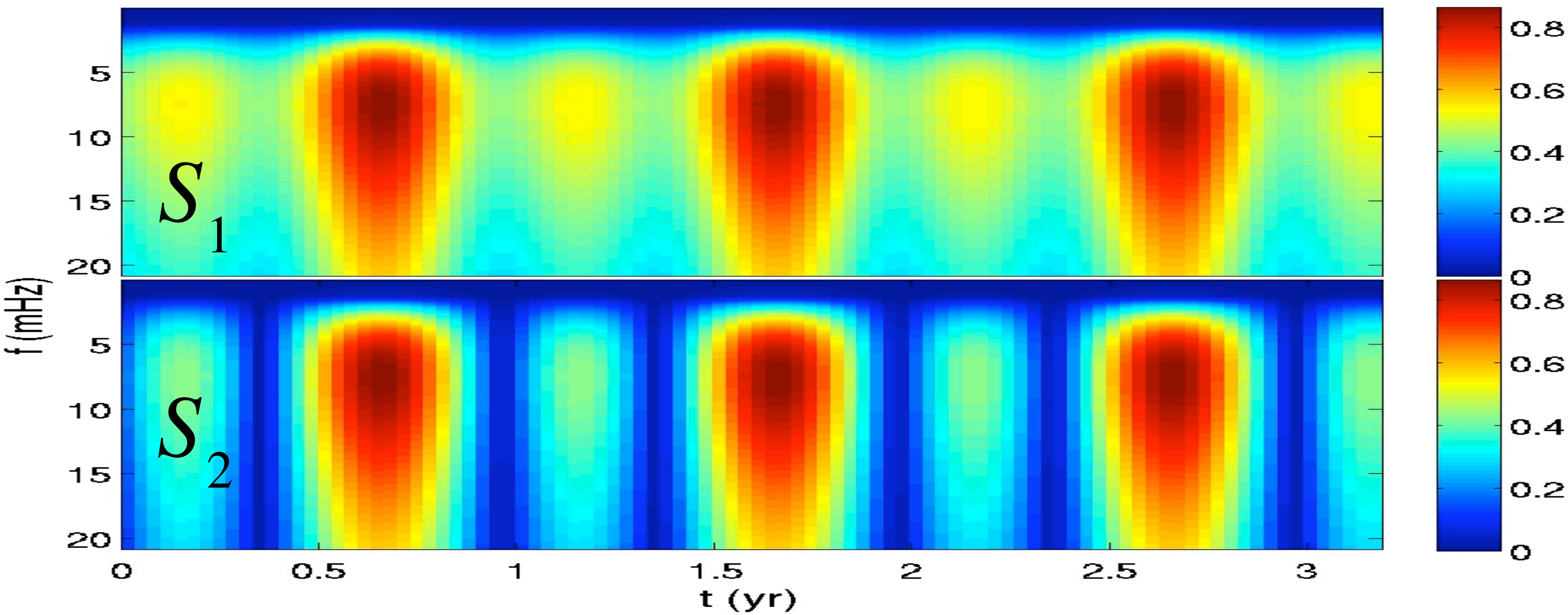
LISA's Directional Sensitivity

Two orthogonal signal components

-for a given sky direction

$$SNR^2 = (A\vec{h} \mid A\vec{h}) = s_1^2 h_1 + s_2^2 h_2$$

$$where\,, \quad \vec{h}^T = (h_{1,} h_2)^T = v^T (h_+, h_x)^T, \qquad v^T v = I$$



See also Rajesh et al (2003)

$$(\phi_s = 60^o, \theta_s = 57^o (Ecliptic))$$